NEUTRINO LUMINOSITY BY THE ORDINARY URCA PROCESS IN AN INTENSE MAGNETIC FIELD

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Abstract. The neutrino luminosity by the ordinary URCA process in a strongly magnetized electron gas is computed. General formulae are presented for the URCA energy loss rates for an arbitrary degree of degeneracy. Analytic expressions are derived for a completely degenerate, relativistic electron plasma in the special case of neutron-proton conversion. Numerical results are given for more general cases.

The main results are as follows: the URCA energy loss rates are drastically reduced for the regime of great degeneracy by a factor up to 10^{-3} for $\Theta \simeq 1$, and $T_9 \lesssim 10$, where $\Theta = H/H_q$, $H_q = m^2c^3/e\hbar = 4.414 \times 10^{13}$ G. In the non-degenerate regime the neutrino luminosity is enhanced approximately linearly with Θ for the temperature range $1 \lesssim T_9 \lesssim 10$. Possible applications to white dwarfs and neutron stars are briefly discussed.

1. Introduction

The theory of weak interaction has been of great interest in astrophysics ever since Fermi introduced his farsighted original theory of beta decay in 1935. In the next few years, astrophysics saw its greatest impact from modern nuclear physics. After the great success of Bethe's theory of the proton-proton chain and the C-N-O cycle for main-sequence stars, Gamow and Schönberg in 1941 introduced the URCA process to astrophysics. In the URCA process, a nucleus alternatively captures an electron and undergoes a beta decay, meanwhile emitting a neutrino and an anti-neutrino. Energy is cyclically carried away by the escaping neutrinos with seemingly no noticeable change for the nuclei participating in the reactions. The characteristic feature for such a cyclic process is analogous to the loss of money in the Casino de URCA, and hence the name.* The various aspects of the URCA process have since been discussed, emphasized and modified by Chiu (1961, 1968), Chiu and Salpeter (1964), Cameron (1959), and Tsuruta and Cameron (Tsuruta, 1964; Tsuruta and Cameron, 1965, 1970). The theory for the modified URCA process has been refined and worked out in detail by Finzi, Bahcall and Wolf (Finzi, 1965; Bahcall and Wolf, 1965; Finzi and Wolf, 1968).

In recent years, there has been considerable interest in the effects of an intense magnetic field (of the order of 10^{13} G) on astrophysical processes. The discovery of the pulsar has greatly accelerated this trend. It is now generally accepted that stellar objects such as white dwarfs and neutron stars are very likely to have a strong magnetic field of the order of $10^8 \sim 10^{13}$ G. The importance of such an ultrastrong magnetic field is obvious in the unified model for pulsars proposed by Chiu and Canuto

* We have been recently informed that in Gamow home-dialect (Odessa dialect) URCA means thief – (Private communication from Prof. G. Wataghin).

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(Chiu and Canuto, 1969, 1970) in connection with radio and optical radiation mechanisms. The possible existence of a powerful internal magnetic field in white dwarfs was first conjectured by Ostriker (1968) to explain the discrepancy between the radius of Sirius B and that predicated from its known mass and the classical mass-radius relation.

The origin for such intense magnetic field is usually attributed to gravitational collapse of the main-sequence stars (Woltjer, 1964); however, a new mechanism called the LOFER (coined from Landau orbital ferromagnetism) has recently been proposed by Lee *et al.* (1969).

Vis-à-vis these latest developments just delineated, we have conducted a systematic reinvestigation of all the neutrino processes of astrophysical interest subjected to a strong magnetic field of order of $H_q \equiv m^2c^3/e\hbar = 4.414 \times 10^{13}$ G. These processes are:

(1) The ordinary and modified URCA process; (2) The plasmon neutrino process; (3) The photo neutrino process; (4) Neutrino bremsstrahlung; and (5) Pair annihilation to neutrinos.

In this paper we shall confine ourselves to studying the effects of a high magnetic field on the simple classical URCA process of Gamow and Schönberg (i.e., the ordinary URCA process). The various subsequent modifications and variations of the original URCA process will not be considered here. These include the modified URCA process originally proposed by Chiu and Salpeter (1964), the nuclear URCA process treated by Hansen (1966, 1968), and the combined thermal and vibrational URCA process considered most recently by Tsuruta and Cameron (1970).

The results for process (2) have been presented in this journal previously (Canuto *et al.*, 1970a, b). The results for process (4) were published elsewhere (Canuto *et al.*, 1970). Processes (3) and (5) are currently under investigation.

In Section 2, basic equations are given. Analytic expressions are derived in Section 3 for various regimes of degeneracy. In Section 4, more general cases are investigated with the use of an electronic computer, for a wide range of temperatures, densities, and magnetic field strengths that are of interest. The URCA energy loss rates in a magnetic field are then compared with the neutrino luminosities in the absence of a magnetic field.

2. The Rate of Energy Loss by the Ordinary URCA Process in a Strongly Magnetized Plasma

In the ordinary URCA process, electron energy is converted into neutrinos via the cyclic process

$$(Z, A) \rightarrow (Z + 1, A) + e^- + \tilde{\nu}_e, \tag{1}$$

$$e^- + (Z + 1, A) \rightarrow (Z, A) + \nu_e. \tag{2}$$

In the absence of an intense magnetic field, the classical URCA process of Gamow and Schönberg is dictated by the electron capture reaction (2) at high densities.

At moderate densities, both electron capture and beta decay contribute to the URCA energy loss rate. The criteria of whether beta decay or electron capture dominates the ordinary URCA process has been discussed by Chiu (1968). In the presence of a strong magnetic field, the cyclic URCA process is drastically affected. To focus our attention on the effects brought about by the magnetic field we make the following set of simplifying assumptions: (1) Nucleons are treated non-relativistically; (2) Orbital quantization of the protons due to the magnetic field is entirely neglected; (3) The Coulomb factor is set equal to unity. We shall first compute the energy loss rate due to beta decay in Part A of this section. The contribution to the URCA process energy loss due to electron capture will be treated in Part B. Physical significance introduced by the magnetic field as well as further assumptions will be pointed out at appropriate stages.

A. BETA DECAY

The beta decay rate in a strong magnetic field has been discussed in considerable detail in a previous paper by Fassio-Canuto (1969). The necessary information for predicating the energy loss rate can be obtained by a trivial generalization of her work. Consequently no derivations are given in this paper. We present only the results.

The vector current conserving $V-A$ weak interaction Hamiltonian for the β decay process is represented by the following Hamiltonian density

$$H_{\text{int}} = \frac{g_v}{\sqrt{2}} \int [\bar{\psi}_P \gamma_\mu (1 + \zeta\gamma_5) \psi_N] [\bar{\psi}_e \gamma_\mu (1 + \gamma_5) \psi_{\nu_e}] \, \mathrm{d}\mathbf{r}, \tag{3}$$

where

$$\begin{aligned} \psi_P &= \Omega_P^{-1/2} e^{iPr/\hbar} U(\mathbf{P}), \\ \psi_N &= \Omega_N^{-1/2} e^{iQr/\hbar} U(\mathbf{Q}), \\ \psi_{\nu_e} &= \Omega_{\nu_e}^{-1/2} e^{iqr/\hbar} U(\mathbf{q}), \\ \psi_e &= e^{-iEt/\hbar} \psi_e(\mathbf{r}). \end{aligned}$$

The wave functions ψ_P, ψ_N, ψ_e, ψ_{ν_e} are referring to the proton, the neutron, the electron and the neutrino respectively. The electron wave function satisfies the Dirac equation in a magnetic field

$$\left[\gamma_\mu \frac{\partial}{\partial x_\mu} + \left(\frac{mc}{\hbar}\right)\right] \psi_e + \frac{ie}{\hbar c} \gamma_\mu A_\mu \psi_e = 0. \tag{4}$$

Assuming that the magnetic field is spatially homogeneous, constant in time and directed along the z-axis, the vector potential A_μ is taken to be $A_x = -yH$, $A_z = A_y = A_0 = 0$. The spinors in a magnetic field $\psi_e(r)$ were given in our previous papers (Canuto and Chiu, 1968a, b, c), and will not be elaborated here.

The energy loss rate is given by

$$l_{\bar{\nu}}(\mathrm{H}) = \frac{\mathcal{N}_Z}{\varrho} \frac{\Theta}{\tau} mc^2 \sum_{n=0}^{N\max} \alpha_n \int_{a_n}^{\Delta} \mathrm{d}\varepsilon \, \frac{\varepsilon (\Delta - \varepsilon)^3}{\sqrt{\varepsilon^2 - a_n^2}} [1 - f(\varepsilon)], \tag{5}$$

where $\mathcal{N}_Z$ is the number density of the parent nuclei (Z, A) ready to decay,

$$\Theta = H/H_q, \quad H_q = m^2c^3/e\hbar = 4.414 \times 10^{13}\ \mathrm{G},$$

$$\frac{1}{\tau} = (1 + 3\zeta^2)\, g_v^2 \left[\frac{m^5c^4}{2\pi^3\hbar^7}\right], \quad \Delta = Q/mc^2,$$

$\zeta \equiv g_A/g_V$, g_V, g_A are vector and axial vector coupling constants in the $V-A$ law respectively.

The quantities $\alpha_n \equiv 1 - \frac{1}{2}\delta_{n,0}$, $a_n^2 \equiv 2n\Theta + 1$, ε is the energy of the electron measured in units of the rest energy of the electron, μ is the chemical potential including rest mass of the electron in units of mc^2, $f(\varepsilon) = [1 + e^{(\varepsilon-\mu)/\phi}]^{-1}$ is the Fermi distribution for the electrons. The orbital quantization of the electrons in a magnetic field is reflected by the finite sum which extends from $n=0$ to $n=N_{\max}$. The maximum quantum number $N_{\max}$ has to be such that $\varepsilon^2 - a_n^2$ is positive. The discontinuities introduced by such imposed restrictions have been discussed in our earlier papers (Canuto and Chiu, 1968a, b, c; Fassio-Canuto, 1969). The significance of the β decay inhibition factor $1-f(\varepsilon)$, (due to the exclusion principle) in astrophysical applications was discussed by Alpher *et al.* (1953) in connection with universe expansion, and was emphasized by Bahcall (1962a, b; 1963) for electron capture in stellar interiors.

B. ELECTRON CAPTURE

The fact that atoms in stellar interiors are highly ionized (due to enormous temperatures encountered) implies that only continuum electron capture is important (Bahcall, 1962a, b, 1964; Finzi and Wolf, 1967). In the presence of a strong magnetic field the electrons are still free to move along the magnetic field. However, the electrons are no longer free but are quantized in the plane perpendicular to the magnetic field. Thus, it is quite clear that the usual concept of the continuum electron capture process is modified by the presence of the magnetic field. The energy loss rate can be obtained by using Fierz's reordering theorem and standard field theoretical methods. The result can be cast into the form

$$l_\nu(\mathrm{H}) = \frac{\mathcal{N}_{Z+1}}{\varrho} \frac{\Theta}{\tau} mc^2 \sum_{n=0}^{N_{\max}} \alpha_n \int_{\Delta}^{\infty} \mathrm{d}\varepsilon \frac{\varepsilon(\varepsilon - \Delta)^3}{\sqrt{\varepsilon^2 - a_n^2}} f(\varepsilon), \tag{6}$$

where $\mathcal{N}_{Z+1}$ is the number density of the nuclei $(Z+1, A)$, $\Delta \equiv |Q|/mc^2$ is the electron capture threshold energy in units of electron rest mass energy ($\Delta = 2.532$ for electron capture by a single proton), and other symbols have already been defined. For further reference, we shall also write down the reaction rates for β decay and electron capture in a magnetic field, namely,

$$\lambda_{\bar{\nu}}(\mathrm{H}) = \frac{\Theta}{\tau} \sum_{n=0}^{N_{\max}} \alpha_n \int_{a_n}^{\Delta} \mathrm{d}\varepsilon \frac{\varepsilon(\Delta - \varepsilon)^2}{\sqrt{\varepsilon^2 - a_n^2}} [1 - f(\varepsilon)], \tag{7}$$

(for β decay)

$$\lambda_\nu(\mathrm{H}) = \frac{\Theta}{\tau} \sum_{n=0}^{N_{\max}} \alpha_n \int_{\Delta}^{\infty} \mathrm{d}\varepsilon \frac{\varepsilon(\varepsilon - \Delta)^2}{\sqrt{\varepsilon^2 - a_n^2}} f(\varepsilon). \tag{8}$$

(for electron capture)

In the classical limit $\Theta \to 0$, it is easy to show that Equations (5), (6), (7) and (8) reduce to

$$l_{\bar{\nu}}(\mathrm{O}) = \frac{\mathcal{N}_Z}{\varrho} \frac{mc^2}{\tau} \int_{1}^{\Delta} \varepsilon \sqrt{\varepsilon^2 - 1} (\Delta - \varepsilon)^3 [1 - f(\varepsilon)] \, \mathrm{d}\varepsilon, \tag{9}$$

$$l_\nu(\mathrm{O}) = \frac{\mathcal{N}_{Z+1}}{\varrho} \frac{mc^2}{\tau} \int_{\Delta}^{\infty} \varepsilon \sqrt{\varepsilon^2 - 1} (\varepsilon - \Delta)^3 f(\varepsilon) \, \mathrm{d}\varepsilon, \tag{10}$$

$$\lambda_{\bar{\nu}}(\mathrm{O}) = \frac{1}{\tau} \int_{1}^{\Delta} \varepsilon \sqrt{\varepsilon^2 - 1} (\Delta - \varepsilon)^2 [1 - f(\varepsilon)] \, \mathrm{d}\varepsilon, \tag{11}$$

$$\lambda_\nu(\mathrm{O}) = \frac{1}{\tau} \int_{\Delta}^{\infty} \varepsilon \sqrt{\varepsilon^2 - 1} (\varepsilon - \Delta)^2 f(\varepsilon) \, \mathrm{d}\varepsilon, \tag{12}$$

respectively (Canuto and Chiu, 1968a, b, c; Fassio-Canuto, 1969).

C. URCA ENERGY LOSS RATE

The total URCA energy loss rate is the sum of the energy loss rates due to β decay and electron capture. The effect of the magnetic field on the URCA process can be seen most clearly by considering the ratio of the energy loss rates with and without the presence of the magnetic field. We obtain from Equations (5) through (12)

$$\begin{aligned} L_{\mathrm{H}} &= l_{\bar{\nu}}(\mathrm{H}) + l_\nu(\mathrm{H}), \quad \text{(with magnetic field } H\text{)} \\ L_{\mathrm{O}} &= l_{\bar{\nu}}(\mathrm{O}) + l_\nu(\mathrm{O}), \quad \text{(without magnetic field)} \end{aligned} \tag{13}$$

$$\frac{L_H}{L_{\mathrm{O}}} = \Theta \times \frac{\left[\frac{\mathcal{N}_Z(\mathrm{H})}{\varrho}\right] \sum_{n=0}^{N_{\max}} \alpha_n \int_{a_n}^{\Delta} \mathrm{d}\varepsilon \frac{\varepsilon(\Delta - \varepsilon)^3}{\sqrt{\varepsilon^2 - a_n^2}} [1 - f(\varepsilon)] + \left[\frac{\mathcal{N}_{Z+1}(\mathrm{H})}{\varrho}\right] \sum_{n=0}^{N_{\max}} \alpha_n \int_{\Delta}^{\infty} \mathrm{d}\varepsilon \frac{\varepsilon(\varepsilon - \Delta)^3}{\sqrt{\varepsilon^2 - a_n^2}} f(\varepsilon)}{\left[\frac{\mathcal{N}_Z(\mathrm{O})}{\varrho}\right] \int_{1}^{\Delta} \mathrm{d}\varepsilon \, \varepsilon \sqrt{\varepsilon^2 - 1} (\Delta - \varepsilon)^3 [1 - f(\varepsilon)] + \left[\frac{\mathcal{N}_{Z+1}(\mathrm{O})}{\varrho}\right] \int_{\Delta}^{\infty} \mathrm{d}\varepsilon \, \varepsilon \sqrt{\varepsilon^2 - 1} (\varepsilon - \Delta)^3 f(\varepsilon)}. \tag{14}$$

The number densities $\mathcal{N}_Z(\mathrm{H})$ and $\mathcal{N}_{Z+1}(\mathrm{H})$ of the parent nuclei are determined by the conditions of statistical equilibrium and charge neutrality, namely

$$\mathcal{N}_Z(\mathrm{H})\,\lambda_{\tilde{\nu}}(\mathrm{H}) = \mathcal{N}_{Z+1}(\mathrm{H})\,\lambda_\nu(\mathrm{H}), \tag{15a}$$

$$\mathcal{N}_Z(\mathrm{H}) + \mathcal{N}_{Z+1}(\mathrm{H}) = \frac{C_Z\varrho}{AM_p}. \tag{15b}$$

From these equations, we obtain immediately

$$\mathcal{N}_Z(\mathrm{H}) = \frac{\lambda_\nu(\mathrm{H})}{\lambda_\nu(\mathrm{H}) + \lambda_{\tilde{\nu}}(\mathrm{H})}\left[\frac{C_Z\varrho}{AM_p}\right], \tag{16a}$$

$$\mathcal{N}_{Z+1}(\mathrm{H}) = \frac{\lambda_{\tilde{\nu}}(\mathrm{H})}{\lambda_\nu(\mathrm{H}) + \lambda_{\tilde{\nu}}(\mathrm{H})}\left[\frac{C_Z\varrho}{AM_p}\right], \tag{16b}$$

where $\lambda_{\tilde{\nu}}(\mathrm{H})$ and $\lambda_\nu(\mathrm{H})$ are given by Equations (7) and (8), respectively. In the absence of the magnetic field, we obtain a similar set of equations

$$\mathcal{N}_Z(\mathrm{O}) = \frac{\lambda_\nu(\mathrm{O})}{\lambda_\nu(\mathrm{O}) + \lambda_{\tilde{\nu}}(\mathrm{O})}\left[\frac{C_Z\varrho}{AM_p}\right], \tag{17a}$$

$$\mathcal{N}_{Z+1}(\mathrm{O}) = \frac{\lambda_{\tilde{\nu}}(\mathrm{O})}{\lambda_\nu(\mathrm{O}) + \lambda_{\tilde{\nu}}(\mathrm{O})}\left[\frac{C_Z\varrho}{AM_p}\right], \tag{17b}$$

where $\lambda_{\tilde{\nu}}(\mathrm{O})$ and $\lambda_\nu(\mathrm{O})$ are now given by Equations (11) and (12), respectively. In the field-free classical limit $\Theta \to 0$ ($\Theta \equiv H/H_q$, $H_q \equiv m^2c^3/e\hbar = 4.414 \times 10^{13}$ G), we have $\lambda_\nu(\mathrm{H}) \to \lambda_\nu(\mathrm{O})$, $\lambda_{\tilde{\nu}}(\mathrm{H}) \to \lambda_{\tilde{\nu}}(\mathrm{O})$; $\mathcal{N}_Z(\mathrm{H}) \to \mathcal{N}_Z(\mathrm{O})$; $\mathcal{N}_{Z+1}(\mathrm{H}) \to \mathcal{N}_{Z+1}(\mathrm{O})$; consequently, $L_\mathrm{H}/L_\mathrm{O}$ becomes unity, as expected. Equation (14) is our main result which we shall discuss in detail in Part 3 and Part 4 of this paper with further simplifying assumptions. It is apparent that our formulation can be generalized to all nuclei which are either in the ground state or in one of the excited states. The case involving excited states without external magnetic field has been extensively discussed by Cameron (1959, 1966), Tsuruta and Cameron (1965), Chiu (1961) and Hansen (1966, 1968).

3. URCA Energy Loss Rate in a Strongly Magnetized Electron Gas Due to Fundamental Weak Interactions – Analytic Approximations

In the absence of a magnetic field, the number densities of the parent nuclei $\mathcal{N}_Z(\mathrm{O})$, and $\mathcal{N}_{Z+1}(\mathrm{O})$ are both coupled to the reaction rates $\lambda_\nu(\mathrm{O})$ and $\lambda_{\tilde{\nu}}(\mathrm{O})$ through the steady state condition and charge neutrality. A similar situation prevails in the presence of the field. More specifically, the number densities of the parent nuclei $\mathcal{N}_Z(\mathrm{H})$, and $\mathcal{N}_{Z+1}(\mathrm{H})$ are still coupled to the reaction rates $\lambda_{\tilde{\nu}}(\mathrm{H})$ and $\lambda_\nu(\mathrm{H})$ through the conditions of statistical equilibrium and charge neutrality (Equation (15)). However, as indicated explicitly, these quantities are now complicated functions of the magnetic field via the field strength parameter Θ. The matter density ϱ is related to the electron number density by the requirement of charge neutrality regardless of the presence of the magnetic field. Nevertheless, the electron number density in a magnetic field is a rather complicated function of Θ (Canuto and Chiu, 1968a)

$$\mathcal{N}_e(\mathrm{H}) = \frac{\Theta}{4\pi^2\lambdabar^3} \sum_{n=0}^{\infty} \sum_{r=1}^{2} \int_{-\infty}^{\infty} \mathrm{d}x \left\{1 + \exp\left[\frac{E(n,x,r) - \mu}{\phi}\right]\right\}^{-1}, \tag{18}$$

where $\phi = kT/mc^2$, $x = p_z/mc$, $\lambdabar = \hbar/mc$, $\Theta = H/H_q$, $H_q = m^2c^3/e\hbar$, $E(x,n,r) = \{1 + x^2 + 2\Theta[n + r - 1]\}^{1/2}$, μ is the chemical potential including electron rest mass in units of mc^2. In the classical limit $\Theta \to 0$ it can be shown that Equation (18) becomes (Canuto and Chiu, 1968a)

$$\mathcal{N}_e(\mathrm{O}) = \frac{1}{\pi^2\lambdabar^3} \int_1^{\infty} \varepsilon\sqrt{\varepsilon^2 - 1}\, f(\varepsilon)\, \mathrm{d}\varepsilon, \tag{19}$$

where $f(\varepsilon) = [1 + e^{(\varepsilon - \mu)/\phi}]^{-1}$ is the usual Fermi distribution, and ε is the energy of the electron in units of mc^2.

The exact integrations involved in Equations (14), (16), (17) and (18) are too complicated to carry out analytically. In order to make further progress we shall therefore concentrate on the simplest URCA process that consists of the fundamental beta decay and electron capture by a single proton. To study the rough behavior of the energy loss rate in a magnetic field, we further assume that the initial number densities of the neutrons are equal to that of the protons. The study of such a drastically simplified situation, although qualitative at best, can shed light toward our understanding of the general trend of the solution, and we devote this section to that purpose. Under these assumptions we obtain from Equations (14), (15) and (16)

$$\frac{L_{\mathrm{H}}}{L_{\mathrm{O}}} = \Theta \frac{\displaystyle\sum_{n=0}^{N_{\max}} \alpha_n \int_{a_n}^{\Delta} \mathrm{d}\varepsilon \frac{\varepsilon(\Delta - \varepsilon)^3}{\sqrt{\varepsilon^2 - a_n^2}} \left[\frac{1}{e^{\eta} e^{-\varepsilon/\phi} + 1}\right] + \sum_{n=0}^{N_{\max}} \alpha_n \int_{\Delta}^{\infty} \mathrm{d}\varepsilon \frac{\varepsilon(\varepsilon - \Delta)^3}{\sqrt{\varepsilon^2 - a_n^2}} \left[\frac{1}{e^{\varepsilon/\phi} e^{-\eta} + 1}\right]}{\displaystyle\int_1^{\Delta} \mathrm{d}\varepsilon \frac{\varepsilon\sqrt{\varepsilon^2 - 1}(\Delta - \varepsilon)^3}{e^{\eta} e^{-\varepsilon/\phi} + 1} + \int_{\Delta}^{\infty} \mathrm{d}\varepsilon \frac{\varepsilon\sqrt{\varepsilon^2 - 1}(\varepsilon - \Delta)^3}{e^{\varepsilon/\varphi} e^{-\eta} + 1}}, \tag{20}$$

and that

$$\mathcal{N}_0(\mathrm{H}) = \mathcal{N}_1(\mathrm{H}) = \mathcal{N}_e(\mathrm{H}), \tag{21}$$

where $\mathcal{N}_e(\mathrm{H})$ is given by Equation (18). The effect of the magnetic field on the electron number density has been shown to be small (Canuto and Chiu, 1968a, b, c). Therefore, it is legitimate to take the field free electron number density. For the case of neutron proton conversion we have

$$\mathcal{N}_0 = \mathcal{N}_1 = \mathcal{N}_e(\mathrm{O}) = \frac{1}{\pi^2\lambdabar^3} \int_1^{\infty} \varepsilon\sqrt{\varepsilon^2 - 1}\, f(\varepsilon)\, \mathrm{d}\varepsilon,$$

where $\mathcal{N}_0$, $\mathcal{N}_1$, and $\mathcal{N}_e$ are the number densities for the neutron, the proton and the electron respectively. We shall now obtain some analytic approximations to Equation (20) for the various regimes of degeneracy based on the number density of a field-free electron gas (Equation (19)).

1. Non degenerate

From Equation (20) we see that for neutron proton conversion

$$\frac{L_{\mathrm{H}}}{L_{\mathrm{O}}} = \Theta \frac{\sum\limits_{n=0}^{N_{\max}} \alpha_n \int\limits_{a_n}^{\Delta} \frac{\varepsilon(\Delta-\varepsilon)^3}{\sqrt{\varepsilon^2 - a_n^2}}\,\mathrm{d}\varepsilon + \sum\limits_{n=0}^{N_{\max}} \alpha_n e^{\eta} \int\limits_{\Delta}^{\infty} \frac{\varepsilon(\varepsilon-\Delta)^3}{\sqrt{\varepsilon^2 - a_n^2}}\, e^{-\varepsilon/\phi}\,\mathrm{d}\varepsilon}{\int\limits_{1}^{\Delta} \varepsilon\sqrt{\varepsilon^2-1}\,(\Delta-\varepsilon)^3\,\mathrm{d}\varepsilon + e^{\eta}\int\limits_{\Delta}^{\infty} \varepsilon\sqrt{\varepsilon^2-1}\,(\varepsilon-\Delta)^3\, e^{-\varepsilon/\phi}\,\mathrm{d}\varepsilon}. \tag{22}$$

The integrals for β decay (i.e., the integral in the first sum of the numerator and the first integral in the denominator) can both be integrated in closed form. However, the integrals for electron capture cannot be integrated in terms of elementary functions. The factor $e^{\eta}\,(\eta=\mu/\phi)$, in the field-free limit are given by Cox (1969) for non-relativistic and extreme relativistic limits.

2. Slightly degenerate ($\eta \ll -1$)

In this case we obtain from Equation (20)

$$\frac{L_{\mathrm{H}}}{L_{\mathrm{O}}} = \Theta$$

$$\times \frac{\sum\limits_{n=0}^{N_{\max}} \alpha_n \int\limits_{a_n}^{\Delta} \mathrm{d}\varepsilon \frac{\varepsilon(\Delta-\varepsilon)^3}{\sqrt{\varepsilon^2-a_n^2}} \sum\limits_{l=0}^{\infty} (-1)^l e^{l\eta} e^{-l\varepsilon/\phi} + \sum\limits_{n=0}^{N_{\max}} \alpha_n \int\limits_{\Delta}^{\infty} \mathrm{d}\varepsilon \frac{\varepsilon(\varepsilon-\Delta)^3}{\sqrt{\varepsilon^2-a_n^2}} e^{\eta} e^{-\varepsilon/\phi}}{\int\limits_{1}^{\Delta} \varepsilon\sqrt{\varepsilon^2-1}\,(\Delta-\varepsilon)^3 \sum\limits_{l=0}^{\infty} (-1)^l e^{l\eta} e^{-l\varepsilon/\phi}\,\mathrm{d}\varepsilon + \int\limits_{\Delta}^{\infty} \varepsilon\sqrt{\varepsilon^2-1}\,(\varepsilon-\Delta)^3 e^{\eta} e^{-\varepsilon/\phi}\,\mathrm{d}\varepsilon}. \tag{23}$$

3. Partially degenerate ($\eta \to 0$)

The ratio $L_{\mathrm{H}}/L_{\mathrm{O}}$ can be written as

$$\frac{L_{\mathrm{H}}}{L_{\mathrm{O}}} = \Theta \frac{\sum\limits_{n=0}^{N_{\max}} \alpha_n \int\limits_{a_n}^{\Delta} \mathrm{d}\varepsilon \frac{\varepsilon(\Delta-\varepsilon)^3}{\sqrt{\varepsilon^2-a_n^2}} \frac{e^{\varepsilon/\phi}}{e^{\varepsilon/\phi}+1} + \sum\limits_{n=0}^{N_{\max}} \alpha_n \int\limits_{\Delta}^{\infty} \mathrm{d}\varepsilon \frac{\varepsilon(\varepsilon-\Delta)^3}{\sqrt{\varepsilon^2-a_n^2}} \frac{1}{e^{\varepsilon/\phi}+1}}{\int\limits_{1}^{\Delta} \mathrm{d}\varepsilon \left[\frac{\varepsilon\sqrt{\varepsilon^2-1}\,(\Delta-\varepsilon)^3\, e^{\varepsilon/\phi}}{e^{\varepsilon/\phi}+1}\right] + \int\limits_{\Delta}^{\infty} \mathrm{d}\varepsilon \left[\frac{\varepsilon\sqrt{\varepsilon^2-1}\,(\varepsilon-\Delta)^3}{e^{\varepsilon/\phi}+1}\right]}. \tag{24}$$

4. Greatly degenerate ($\eta \gg 1$)

$$\frac{L_{\mathrm{H}}}{L_{\mathrm{O}}} = \Theta$$

$$\times \frac{\sum_{n=0}^{N_{\max}} \alpha_n e^{-\eta} \int_{a_n}^{\Delta} \mathrm{d}\varepsilon \frac{\varepsilon(\Delta-\varepsilon)^3}{\sqrt{\varepsilon^2 - a_n^2}} e^{\varepsilon/\phi} + \sum_{n=0}^{N_{\max}} \alpha_n \sum_{l=0}^{\infty} (-1)^l e^{-l\eta} \int_{\Delta}^{\infty} \mathrm{d}\varepsilon \frac{\varepsilon(\varepsilon-\Delta)^3}{\sqrt{\varepsilon^2 - a_n^2}} e^{l\varepsilon/\phi}}{e^{-\eta} \int_{1}^{\Delta} \varepsilon \sqrt{\varepsilon^2 - 1}\,(\Delta-\varepsilon)^3 e^{\varepsilon/\phi}\,\mathrm{d}\varepsilon + \sum_{l=0}^{\infty} (-1)^l e^{-l\eta} \int_{\Delta}^{\infty} \varepsilon \sqrt{\varepsilon^2-1}\,(\varepsilon-\Delta)^3 e^{l\varepsilon/\phi}\,\mathrm{d}\varepsilon}. \tag{25}$$

5. Completely degenerate ($\eta \to \infty$)

The regime of complete degeneracy is of special interest because the possible applications to neutron stars and white dwarfs. Fortunately this is also the case for which the integrals involved in Equation (20) can be integrated exactly. The integrations are elementary and straight forward although slightly tedious. The result can be cast into the form:

$$\frac{L_{\mathrm{H}}}{L_{\mathrm{O}}} = \Theta \left[\frac{\mathscr{I}(x, y)}{\mathscr{F}(\Delta, \mu)}\right], \quad a_n^2 = 2n\Theta + 1, \tag{26}$$

$$\mathscr{I}(x, y) = \sum_{n=0}^{N_{\max}} \alpha_n a_n^4 \zeta(x, y), \quad x = \Delta/a_n, \quad y = \mu/a_n,$$

$$\zeta(x, y) = \left[\tfrac{3}{8}(4x^2+1)\right] \ln \left[\frac{y + \sqrt{y^2-1}}{x + \sqrt{x^2-1}}\right] + \frac{x}{8}\sqrt{x^2-1}\,(2x^2+13)$$
$$+ \sqrt{y^2-1}\left\{\tfrac{1}{4} y (y^2 + \tfrac{3}{2}) + xy(\tfrac{3}{2}x - y) - x(x^2+2)\right\};$$

$$\mathscr{F}(\Delta, \mu) = \left[\tfrac{1}{16}(6\Delta^2+1)\right] \ln \left[\frac{\Delta + \sqrt{\Delta^2-1}}{\mu + \sqrt{\mu^2-1}}\right] + \sqrt{\Delta^2-1}\,\varphi(\Delta)$$
$$+ \sqrt{\mu^2-1}\,\psi(\Delta, \mu),$$

$$\psi(\Delta, \mu) = \tfrac{1}{6}\mu^5 - \tfrac{3}{5}\Delta\mu^4 + (\tfrac{3}{4}\Delta^2 - \tfrac{1}{24})\mu^3 + (\tfrac{1}{5}\Delta - \tfrac{1}{3}\Delta^3)\mu^2$$
$$- (\tfrac{3}{8}\Delta^2 + \tfrac{1}{16})\mu + (\tfrac{1}{3}\Delta^3 + \tfrac{2}{5}\Delta),$$

$$\varphi(\Delta) = \tfrac{1}{60}\Delta\left[\Delta^4 - 7\Delta^2 - (\tfrac{9}{2})^2\right]; \quad \mu^2 - 1 = \varrho_6^{2/3}.$$

4. Numerical Solutions

In this section we present the numerical results (for the ratio of the URCA energy loss rates) for the special case of the fundamental weak interactions with equal initial population for neutrons and protons. There is no difficulty in principle to integrate numerically the general exact equation (Equation (14)) without introducing the simplifying assumption for the equality of the initial population of the parent nuclei. Nevertheless, it is rather a time consuming task for the computer and we shall confine ourselves to studying this special case. On the other hand, if we are content with the rough behavior of the URCA process in a magnetic field, then the study of the special case for neutron proton conversion is sufficient. To show the general trend of the solutions for more complex nuclei we carried out the integrations of Equation (20) on the IBM 360-95 computer for sample cases of $\Delta = 1.2$, to $\Delta = 10$ at step 0.8 (which covers the threshold energy of up to about 5 MeV). The temperatures range from $\sim 10^8$ K to 10^{10} K, and the field strength parameter Θ ($\Theta = H/H_q$, $H_q = m^2c^3/e\hbar = 4.414 \times 10^{13}$ G) covered are from 10^{-2} to 10^2. The various regimes of degeneracy are covered by the degeneracy parameter η ($\eta = \mu/\phi$) from -10 (non degenerate) to 100 (great degeneracy). In these calculations, the coulomb factor were set equal to unity and we have neglected the orbital quantization of the protons due to the magnetic field. Consequently the effect of the magnetic field on the Coulomb factor

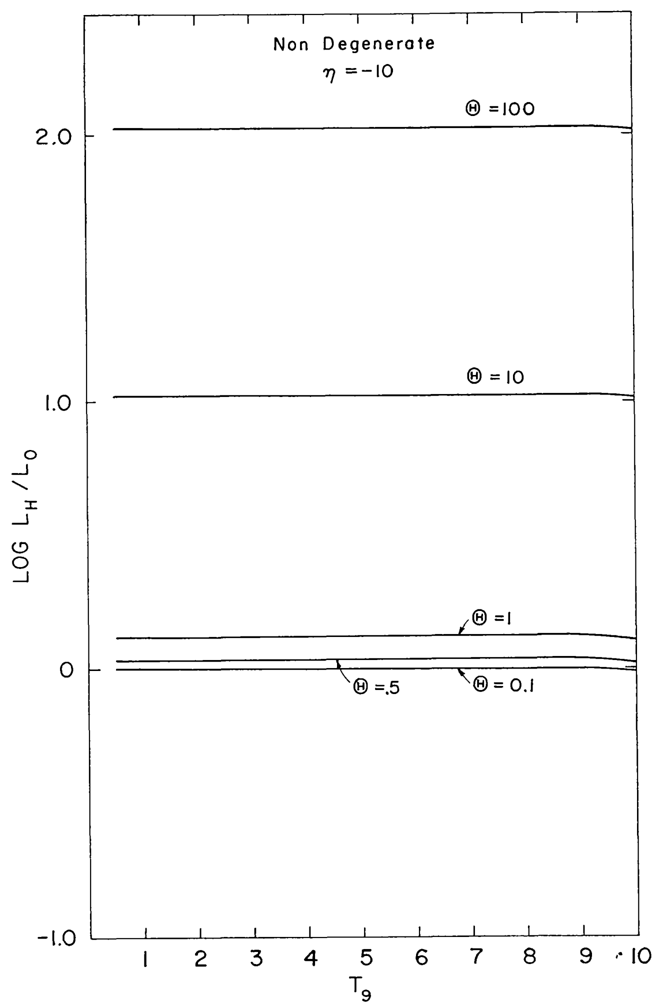

Fig. 1. The ratio of the URCA energy loss rates L_H/L_O as a function of temperature T_9 ($T_9 = 10^{-9}\,T$K) and the field strength parameter Θ ($\Theta = H/H_q$, $H_q = m^2c^3/e\hbar = 4.414 \times 10^{13}$ G). In the non-degenerate regime ($\eta = -10$) with $\Delta = 2.532$. η is the parameter of degeneracy, $\Delta = (M_n - M_p/m)$ is neutron proton mass difference in units of mc^2.

can be neglected. It follows that the ratio L_H/L_O will not be substantially changed even if the coulomb factors were retained in the integrals (with and without the presence of the magnetic field) involved in the ratio L_H/L_O. The approximation of neglecting the coulomb factor is a particularly good one for neutron proton conversion. Hence we have studied this case in greater detail.

The results are summarized in Figures 1–4. In Figure 1, the behavior of $\log L_H/L_O$ as a function of T_9 and Θ is shown for the non-degenerate regime ($\eta = -10$), $\Delta = 2.532$ (neutron proton conversion). We note that for sufficiently low fields ($0.1 \lesssim \Theta \lesssim 1$) the URCA energy loss rates are not significantly changed by the magnetic field. However, at relatively high fields ($\Theta \gg 1$) the ratio L_H/L_O increases approximately linearly. For instance $L_H/L_O \to 100$ for $\Theta \to 100$. The ratio L_H/L_O are rather insensitive to temperature changes in the range $1 \lesssim T_9 \lesssim 9$ for all fields. In Figure 2, $\log_{10}(L_H/L_O)$ as a function of T_9 and Θ is shown for the regime of partial degeneracy with $\eta = 0$, $\Delta = 2.532$. We note that for magnetic fields close to the critical field strength H_q ($\Theta \lesssim 1$), the URCA energy loss rates are decreased by a factor of up to about $10^{-1.5}$ at $T_9 \simeq 10$.

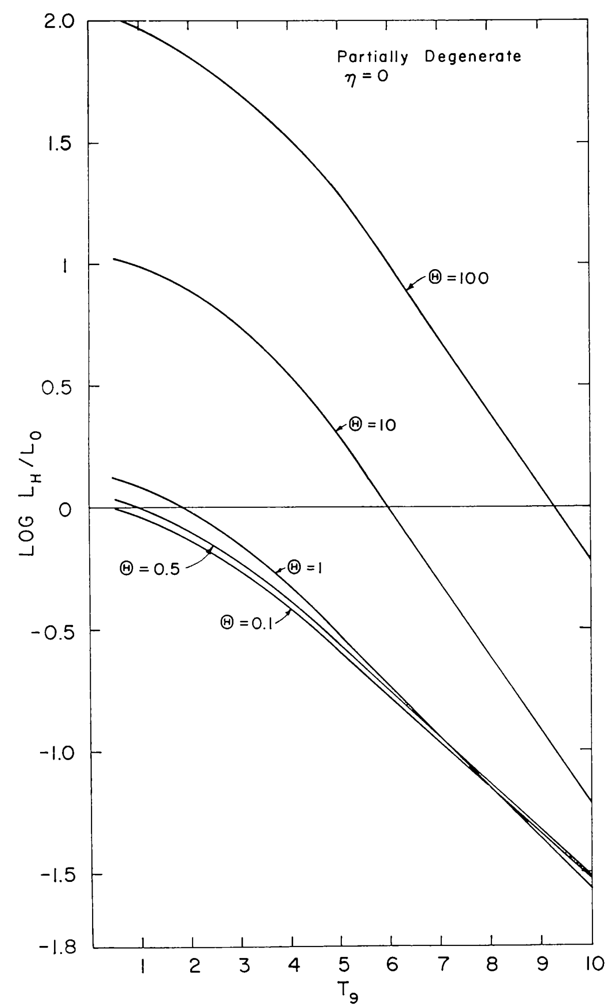

Fig. 2. Same as Figure 1 for the regime of partial degeneracy with $\eta = 0$.

The URCA energy loss rates are very sensitive functions of temperature and the field strength parameter Θ, for instance the URCA energy loss rates are increased by a factor of up to about 100 at $T_9 \simeq 1$, $\Theta = 100$. Thus, the URCA energy loss rates may either be increased or decreased depending on the combination of temperatures, field strengths and most strongly on the degree of degeneracy. The general trend is that the URCA energy losses are greatly decreased in the region of strong degeneracy, and that the energy loss rates are significantly increased for the non-degenerate regime. These general predications can clearly be seen from Figure 3 ($\eta = 20$) and Figure 4 ($\eta = 100$). Our results for the case of complete degeneracy are presented in Table V. It is seen that the neutrino luminosities are in general decreased at very high densities. For instance $L_H/L_O \simeq 3.5 \times 10^{-2}$ at $\varrho_6 = 10^3$, and $\Theta = 1$.

The rough behavior of L_H/L_O as a function of the threshold energy Δ for more complex nuclei (based on the assumption of equal population of the parent nuclei) are summarized in Tables VI–XIII. In the regime of non-degeneracy (Tables VI, VII, and VIII) the URCA energy loss rates are not substantially affected by the magnetic

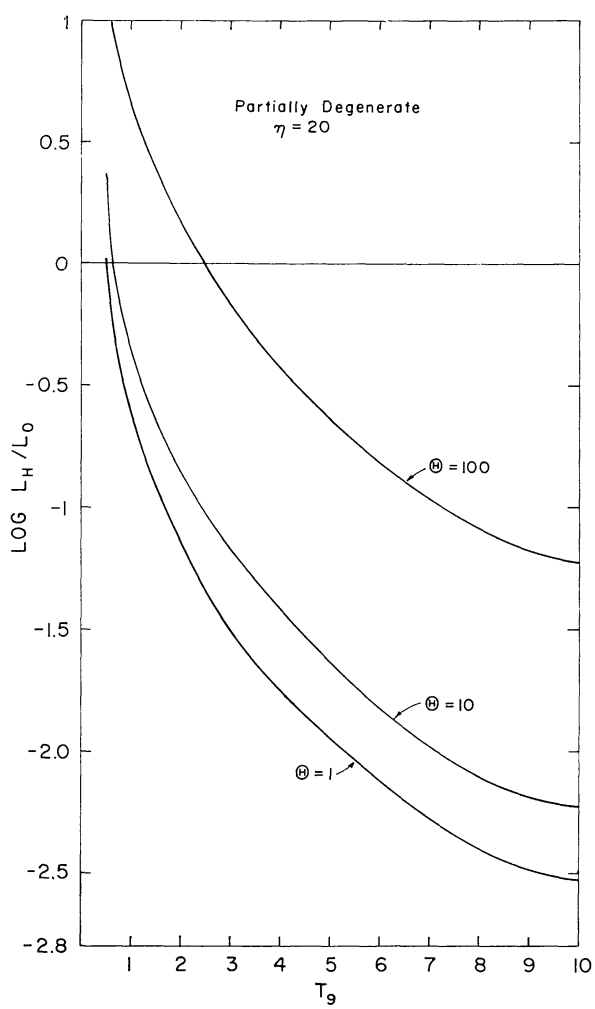

Fig. 3. Same as Figure 1 for $\eta = 20$.

field except at very low threshold energies such as at $\Delta = 1.2$. On the other hand, the URCA energy loss rates are in general drastically decreased in the regime of great degeneracy (Tables IX–XIII) by a factor of up to about 10^{-5} for $\Theta = 0.1$ $T_9 = 10$ at $\Delta = 1.2$. However, the neutrino luminosities are practically unaffected by the field for high threshold energies and at relatively low temperatures ($T_9 = 0.5$, $\Delta = 4.4 \sim 10$, see Table IX).

Ostriker (1968) has conjectured that magnetic fields of the order of 10^8 G may exist in the interiors of white dwarfs. If fields as high as 10^{11} G were possible, then our results for the completely degenerate regime show that the URCA energy loss rates would be reduced by a factor of 10 for $\varrho_6 \sim 0.1$ and by a factor of 10^2 for $\varrho_6 \sim 1000$ (Table V, $\Theta = 0.01$). Thus, the hypothetical presence of high magnetic fields of order of 10^{11} G in white dwarfs would make it much more stable against the URCA process.

The LOFER mechanism proposed by Lee *et al.* (1969) may provide intense magnetic fields of the order of 10^{12} G to 10^{13} G in the interiors of neutron stars. Consequently

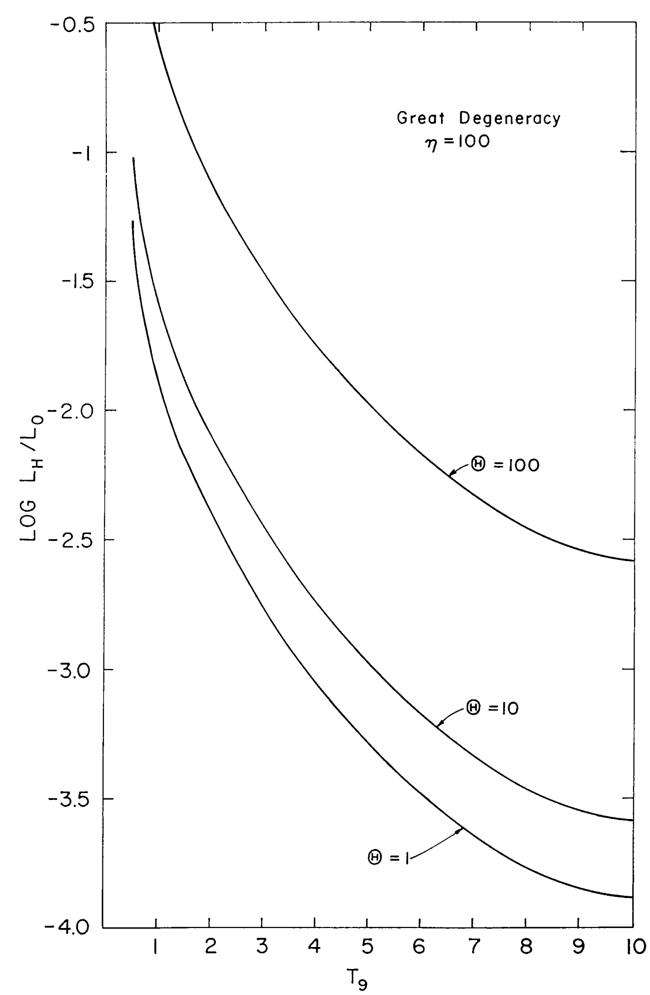

Fig. 4. Same as Figure 1 for the regime of large degeneracy with $\eta = 100$.

TABLE I

The ratio of the URCA energy loss rates $L_{\mathrm{H}}/L_{\mathrm{O}}$ as a function temperature T_9 ($T_9 = 10^{-9}\,T$K) and the field strength parameter Θ ($\Theta = H/H_q$, $H_q = m^2c^3/e\hbar = 4.414 \times 10^{13}$ G) with $\eta = -10$, $\Delta = 2.532$. The other symbols are defined in the text

$\eta = -10$, $L_{\mathrm{H}}/L_{\mathrm{O}}$

Θ	$T_9 = 0.5$	$T_9 = 1$	$T_9 = 5$	$T_9 = 10$
10^{-3}	1.00	1.00	1.00	0.97
10^{-2}	1.00	1.00	1.00	0.97
10^{-1}	1.00	1.00	1.00	0.98
5×10^{-1}	1.08	1.08	1.08	1.05
1	1.31	1.31	1.32	1.28
10	10.45	10.45	10.46	10.15
10^2	104.50	104.50	104.60	101.50

TABLE II

Same as Table I with $\eta = 0$.

$\eta = 0$, $L_{\mathrm{H}}/L_{\mathrm{O}}$

Θ	$T_9 = 0.5$	$T_9 = 5$	$T_9 = 10$
10^{-1}	0.002	-0.594	-1.52
5×10^{-1}	0.032	-0.570	-1.52
1	0.12	-0.524	-1.56
10	1.02	$+0.298$	-1.21
10^2	2.02	$+1.298$	-0.21

TABLE III

Same as Table I with $\eta = 20$

$\eta = 20$, $\log_{10}(L_{\mathrm{H}}/L_{\mathrm{O}})$

Θ	$T_9 = 0.5$	$T_9 = 1$	$T_9 = 5$	$T_9 = 10$
10^{-2}	0	-0.553	-1.90	-2.49
10^{-1}	-0.001	-0.545	-1.91	-2.49
5×10^{-1}	-0.002	-0.545	-1.91	-2.49
1	$+0.023$	-0.597	-1.94	-2.53
10	0.37	-0.351	-1.64	-2.23
10^2	1.37	$+0.644$	-0.64	-1.23

TABLE IV

Same as Table I with $\eta = 100$

$\eta = 100$, $\log_{10}(L_{\mathrm{H}}/L_{\mathrm{O}})$

Θ	$T_9 = 0.5$	$T_9 = 1$	$T_9 = 5$	$T_9 = 10$
10^{-2}	-1.28	-1.87	-3.25	-3.85
10^{-1}	-1.27	-1.86	-3.24	-3.84
5×10^{-1}	-1.27	-1.86	-3.24	-3.84
1	-1.31	-1.90	-3.28	-3.88
10	-1.02	-1.60	-2.98	-3.58
10^2	-0.02	-0.60	-1.98	-2.58

TABLE V

The ratio of the URCA energy lose rates L_H/L_O as a function of density ϱ_6 in g/cm^3 (where $\varrho_6 = 10^{-6}\ \varrho/\mu_e$, μ_e is the molecular weight per electron) and the field strength parameter Θ for a completely degenerate electron gas calculated directly from Equation (26) of Section 3 of the text

L_H/L_O, $\eta = \infty$

ϱ_6	$\Theta = 0.01$	$\Theta = 0.1$	$\Theta = 0.5$	$\Theta = 1.0$
0.01	0.043			
0.10	0.139	0.217		
1.00	0.399	0.476	0.867	
12.00	1.497	1.560	1.630	1.145
14.00	0.785	0.821	0.799	0.637
20.00	0.530	0.545	0.542	0.464
30.00	0.385	0.394	0.393	0.344
50.00	0.268	0.273	0.273	0.243
80.00	0.195	0.199	0.200	0.178
200.00	0.107	0.109	0.109	0.098
500.00	0.058	0.060	0.060	0.054
800.00	0.043	0.044	0.044	0.040
1000.00	0.037	0.038	0.038	0.035

TABLE VI

The ratio of the neutrino luminosities L_H/L_O as a function of the electron capture threshold energy Δ (in units of mc^2) for $\Theta = 0.1$, 0.5 and 1.0; at $T_9 = 0.5$; $\eta = -8$ (non degenerate)

L_H/L_O $\eta = -8$, $T_9 = 0.5$

Δ	$\Theta = 0.1$	$\Theta = 0.5$	$\Theta = 1.0$
1.2	1.30	5.36	10.72
2.0	1.01	1.23	1.83
2.8	1.00	1.06	1.22
3.6	1.00	1.02	1.09
4.4	1.00	1.01	1.04
5.2	1.00	1.01	1.02
6.0	1.00	1.00	1.02
6.8	1.00	1.00	1.01
7.6	1.00	1.00	1.01
8.4	1.00	1.00	1.01
9.2	1.00	1.00	1.00
10.0	1.00	1.00	1.00

TABLE VII

Same as Table VI at $T_9 = 1.0$

L_H/L_O $\eta = -8$, $T_9 = 1.0$

Δ	$\Theta = 0.1$	$\Theta = 0.5$	$\Theta = 1.0$
1.2	1.30	5.36	10.72
2.0	1.01	1.23	1.83
2.8	1.00	1.06	1.22
3.6	1.00	1.02	1.09
4.4	1.00	1.01	1.04
5.2	1.00	1.01	1.02
6.0	1.00	1.00	1.02
6.8	1.00	1.00	1.01
7.6	1.00	1.00	1.01
8.4	1.00	1.00	1.00
9.2	1.00	1.00	1.00
10.0	1.00	1.00	1.00

TABLE VIII

Same as Table VI for $\Theta = 0.1$, 1.0; at $T_9 = 10$ and $\eta = -10$ (non degenerate)

L_H/L_O $\eta = -10$, $T_9 = 10$

Δ	$\Theta = 0.1$	$\Theta = 1.0$
1.2	0.518×10^{-2}	0.215×10^{-1}
2.0	0.782	1.410
2.8	0.992	1.202
3.6	1.000	1.085
4.4	1.000	1.043
5.2	1.000	1.024
6.0	1.000	1.025
6.8	1.000	1.010
7.6	1.000	1.010
8.4	1.000	1.010
9.2	1.000	1.000
10.0	1.000	1.000

TABLE IX

Same as Table VI at $T_9 = 0.5$, $\eta = 50$ (strongly degenerate)

L_H/L_O $\eta = 50$, $T_9 = 0.5$

Δ	$\Theta = 0.1$	$\Theta = 0.5$	$\Theta = 1.0$
1.2	2.03×10^{-2}	2.01×10^{-2}	4.02×10^{-2}
2.0	1.22×10^{-1}	9.66×10^{-2}	1.17×10^{-1}
2.8	2.68×10^{-1}	2.50×10^{-1}	2.72×10^{-1}
3.6	4.68×10^{-1}	4.45×10^{-1}	4.17×10^{-1}
4.4	1.00	1.00	1.00
5.2	1.00	1.00	1.00
6.0	1.00	1.00	1.00
6.8	1.00	1.00	1.00
7.6	1.00	1.00	1.00
8.4	1.00	1.00	1.00
9.2	1.00	1.00	1.00
10.0	1.00	1.00	1.00

TABLE X

Same as Table VI at $T_9 = 1.0$, $\eta = 50$ (strongly degenerate)

L_H/L_O $\eta = 50$, $T_9 = 1.0$

Δ	$\Theta = 0.1$	$\Theta = 0.5$	$\Theta = 1.0$
1.2	0.504×10^{-2}	0.503×10^{-2}	0.101×10^{-1}
2.0	0.304×10^{-1}	0.244×10^{-1}	0.294×10^{-1}
2.8	0.663×10^{-1}	0.062	0.672×10^{-1}
3.6	0.112	0.108	0.103
4.4	0.171	0.173	0.178
5.2	0.243	0.248	0.253
6.0	0.329	0.321	0.327
6.8	0.428	0.431	0.424
7.6	0.539	0.536	0.543
8.4	0.748	0.747	0.742
9.2	1.010	1.004	1.004
10.0	1.010	1.005	1.002

TABLE XI

Same as Table VI at $T_9 = 10.0$. $\eta = 50$ (strongly degenerate)

L_H/L_O $\eta = 50$, $T_9 = 10$

Δ	$\Theta = 0.1$	$\Theta = 0.5$	$\Theta = 1.0$
1.2	0.519×10^{-4}	0.519×10^{-4}	0.104×10^{-3}
2.0	0.320×10^{-3}	0.258×10^{-3}	0.310×10^{-3}
2.8	0.710×10^{-3}	0.670×10^{-3}	0.721×10^{-3}
3.6	0.122×10^{-2}	0.118×10^{-2}	0.113×10^{-2}
4.4	0.187×10^{-2}	0.189×10^{-2}	0.194×10^{-2}
5.2	0.266×10^{-2}	0.270×10^{-2}	0.275×10^{-2}
6.0	0.356×10^{-2}	0.350×10^{-2}	0.355×10^{-2}
6.8	0.458×10^{-2}	0.460×10^{-2}	0.455×10^{-2}
7.6	0.572×10^{-2}	0.570×10^{-2}	0.575×10^{-2}
8.4	0.699×10^{-2}	0.699×10^{-2}	0.694×10^{-2}
9.2	0.839×10^{-2}	0.837×10^{-2}	0.832×10^{-2}
10.0	0.990×10^{-2}	0.984×10^{-2}	0.989×10^{-2}

TABLE XII

Same as Table VI for $\Theta = 0.5$, $\Theta = 1.0$; at $T_9 = 1.0$ and $\eta = 100$ (great degeneracy)

L_H/L_O $\eta = 100$, $T_9 = 1.0$

Δ	$\Theta = 0.5$	$\Theta = 1.0$
1.2	1.29×10^{-3}	2.57×10^{-3}
2.0	6.32×10^{-3}	7.59×10^{-3}
2.8	1.62×10^{-2}	1.75×10^{-2}
3.6	2.83×10^{-2}	2.71×10^{-2}
4.4	4.51×10^{-2}	4.64×10^{-2}
5.2	6.40×10^{-2}	6.53×10^{-2}
6.0	8.20×10^{-2}	8.38×10^{-2}
6.8	1.08×10^{-1}	1.07×10^{-1}
7.6	1.34×10^{-1}	1.35×10^{-1}
8.4	1.64×10^{-1}	1.63×10^{-1}
9.2	1.96×10^{-1}	1.95×10^{-1}
10.0	2.32×10^{-1}	2.33×10^{-1}

TABLE XIII

Same as Table VI for $\Theta = 0.1$, $\Theta = 1.0$; at $T_9 = 10.0$ and $\eta = 100$ (great degeneracy).

L_H/L_O $\eta = 100$, $T_9 = 10.0$

Δ	$\Theta = 0.1$	$\Theta = 1.0$
1.2	0.131×10^{-4}	0.262×10^{-4}
2.0	0.812×10^{-4}	0.785×10^{-4}
2.8	0.180×10^{-3}	0.183×10^{-3}
3.6	0.310×10^{-3}	0.287×10^{-3}
4.4	0.476×10^{-3}	0.495×10^{-3}
5.2	0.678×10^{-3}	0.702×10^{-3}
6.0	0.911×10^{-3}	0.908×10^{-3}
6.8	1.173×10^{-3}	1.165×10^{-3}
7.6	1.466×10^{-3}	1.473×10^{-3}
8.4	1.794×10^{-3}	1.781×10^{-3}
9.2	2.156×10^{-3}	2.138×10^{-3}
10.0		2.546×10^{-3}

our results for the regime of great degeneracy show that the neutrino luminosities by the URCA process are also diminished by a factor of up to about 100 for densities range from $\varrho = 10^8 \sim 10^9$.

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References

Alpher, R. A., Follin, J. A., and Herman, R. C.: 1953, 'Physical Conditions in the Initial Stages of the Expanding Universe', *Phys. Rev.* **96**, 1347.

Bahcall, J. N.: 1962a, *Phys. Rev.* **126**, 1143.

Bahcall, J. N.: 1962b, *Phys. Rev.* **128**, 1297.

Bahcall, J. N.: 1964, 'Electron Capture in Stellar Interiors', *Astrophys. J.* **139**, 318.

Bahcall, J. N. and Wolf, R. A.: 1965a, 'Neutron Stars. I: Properties at Absolute Zero Temperatures', *Phys. Rev.* **140**, B1445.

Bahcall, J. N. and Wolf, R. A.: 1965b, 'Neutron Stars. II: Neutrino-Cooling and Observability', *Phys. Rev.* **140**, B1452.

Cameron, A. G. W.: 1959, 'Photobeta Reactions in Stellar Interiors', *Astrophys. J.* **130**, 452.

Cameron, A. G. W.: 1966, *Nuclear Astrophysics*, second edition (compiled by Edelstein, S., Langer, W., and Watkins, D. W.).

Cameron, A. G. W.: 1969, 'Neutron Stars', *Ann. Rev. Astron. Astrophys.*, to be published.

Canuto, V. and Chiu, H. Y.: 1968a, 'Quantum Theory of an Electron Gas in Intense Magnetic Field', *Phys. Rev.* **173**, 1210.

Canuto, V. and Chiu, H. Y.: 1968b, 'Thermodynamic Properties of a Magnetized Fermi Gas', *Phys. Rev.* **173**, 1220.

Canuto, V. and Chiu, H. Y.: 1968c, 'The Magnetic Moment of a Magnetized Fermi Gas', *Phys. Rev.* **173**, 1229.

Canuto, V., Chiu, H. Y., Chou, C. K., and Fassio-Canuto. L.: 1970, 'Neutrino Bremsstrahlung in an Intense Magnetic Field', *Phys. Rev.*, in press.

Canuto, V., Chiuderi, C., and Chou, C. K.: 1970a, 'Plasmon Neutrinos Emission in a Strong Magnetic Field. I: Transverse Plasmons', *Astrophys. Space Sci.* **7**, 407.

Canuto, V., Chiuderi, C., and Chou, C. K.: 1970b, 'Plasmon Neutrinos Emission in a Strong Magnetic Field. II: Longitudinal Plasmons', *Astrophys. Space Sci.*, in press.

Canuto, V., Chiu, H. Y., and Fassio-Canuto, L.: 1969, 'Electron Bremsstrahlung in Intense Magnetic Fields', *Phys. Rev.* **185**, 1607.

Chiu, H. Y.: 1961, 'Neutrino Emission Process, Stellar Evolution and Supernovae, Part I', *Ann. Phys.* **15**, 1.

Chiu, H. Y.: 1968, *Stellar Physics*, Volume I, Chapter 6, Blaisdell Publishing Company.

Chiu, H. Y. and Canuto, V.: 1969, 'Radio Emission from Magnetic Neutron Stars', *Phys. Rev. Letters* **22**, 415.

Chiu, H. Y. and Canuto, V.: 1970, 'Theory of Radiation Mechanisms for Pulsars I', *Astrophys. J.*, to be published.

Cox, J. P. and Giuli, R. T.: 1968, *Principles of Stellar Structure*, Volume 2, Chapter 24, Gordon and Breach.

Fassio-Canuto, L.: 1969, 'Neutron Beta Decay in a Strong Magnetic Field', *Phys. Rev.* **187**, 2141.
Finzi, A.: 'Cooling of a Neutron Star by the URCA Process', *Phys. Rev.* **137**, B472.
Finzi, A. and Wolf, R. A.: 1967, 'Type I Supernovae', *Astrophys. J.* **150**, 115.
Finzi, A. and Wolf, R. A.: 1968, 'Hot, Vibrating Neutron Stars', *Astrophys. J.* **153**, 835.
Gamow, G. and Schönberg, M.: 1941, *Phys. Rev.* **59**, 539.
Hansen, C. J.: 1966, Ph.D. Thesis, Yale University, unpublished.
Hansen, C. J.: 1968, *Astrophys. Space Sci.* **1**, 499.
Lee, H. J., Canuto, V., Chiu, H. Y., and Chiuderi, C.: 1969, 'New State of Ferromagnetism in Degenerate Electron Gas and Magnetic Fields in Collapsed Bodies', *Phys. Rev. Letters* **23**, 390.
Ostriker, J. P. and Hartwick, F. D. A.: 1968, 'Rapidly Rotating Stars. IV: Magnetic White Dwarfs', *Astrophys. J.* **153**, 797.
Tsuruta, S.: 1964, Ph.D. Thesis, Columbia University (unpublished).
Tsuruta, S. and Cameron, A. G. W.: 1965, *Can. J. Phys.* **43**, 2056.
Tsuruta, S. and Cameron, A. G. W.: 1970, 'URCA Shells in Dense Stellar Interiors', *Astrophys. Space Sci.*, in press.
Woltjer, L.: 1964, *Astrophys. J.* **140**, 1309.